

Module 05

Regression Analysis

* Regression:-

Regression is the measure of the average relationship between two or more variables in terms of the original units of the data.

* Techniques of Regression:-

1. Regression Equations:- are algebraic expressions of the regression lines. Since there are two regression lines, there are two regression equations - the regression equation of x on y is used to describe the variations in the values of x for given changes in y and the regression equation of y on x is used to describe the variation in the values of y for given changes in x .

2. Regression Lines:-

Regression line refers to describe the average relationship between the two variable, say x and y . It reveals mean value of y . The equation of regression line is known as "Regression equation".



3. Regression Coefficient :-

The rate of change of variable for unit change in the other variable is called the regression co-efficient of former on the latter. Since there are two regression lines there are two regression co-efficients.

- Regression co-efficient of (x on y) is denoted by :

$$b_{xy} = \frac{\sum xy}{\sum y^2}$$

- (y on x) is denoted by :

$$b_{yx} = \frac{\sum xy}{\sum x^2}$$

* Standard Error of estimate

Standard error of estimate is the measure the variation around the computed regression line.

* Problems

1. Compute 2 regression co-efficient when $r = 0.8$
 $\sum x = 5$ $\sum y = 7$

Sol:- $b_{xy} = r \frac{\sum x}{\sum y} = 0.8 \left(\frac{5}{7} \right) = 0.8 \times 0.71 = 0.568$ (29)
0.57



$$b_{yx} = \frac{\sum xy}{\sum x} = \frac{0.8 \times \frac{1}{5}}{1.4} = 0.8 \times 1.4 = 1.12$$

2. Calculate 2 Regression coefficient when

$$r = -0.99$$

$$\sum x = 10$$

$$\sum y = 12$$

Sol:- $b_{xy} = r \frac{\sum x}{\sum y} = -0.99 \left(\frac{10}{12} \right) = -0.99 \times 0.833 = -0.82$

$$b_{yx} = r \frac{\sum y}{\sum x} = -0.99 \left(\frac{12}{10} \right) = -0.99 \times 1.2 = -1.188$$

3. $b_{xy} = 0.8$

$$b_{yx} = 0.6$$

Find r

Sol:- $r = \sqrt{b_{xy} \times b_{yx}} = \sqrt{0.8 \times 0.6} = \sqrt{0.48} = 0.69$

4. Find r .

$$b_{xy} = 0.57 \quad b_{yx} = 1.12$$

Sol:- $r = \sqrt{b_{xy} \times b_{yx}} = \sqrt{0.57 \times 1.12} = \sqrt{0.63} = 0.79$

5. If $b_{xy} = 1.36$ $b_{yx} = 0.6132$ Find r .

Sol:- $r = \sqrt{b_{xy} \times b_{yx}} = \sqrt{1.36 \times 0.6132} = \sqrt{0.83} = 0.91$

6. If $b_{xy} = -0.36$ $b_{yx} = -1.38$ Find r .

Sol:- $r = \sqrt{b_{xy} \times b_{yx}} = \sqrt{-0.36 \times -1.38} = \sqrt{0.49} = 0.70$



7. obtain the 2 regression equations from the following.

Particulars	x	y
Mean	20	25
variance	4	9

co-efficient - 0.75
of correlation

Sol:- The regression equation x on y

$$(x - \bar{x}) = r \frac{\sigma_x}{\sigma_y} (y - \bar{y})$$

$$r = \frac{\sigma_x}{\sigma_y} = 0.75 \times \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$$

$$(x - 20) = 0.75 \times \frac{2}{3} (y - 25)$$

$$(x - 20) = 0.5 (y - 25)$$

$$x - 20 = 0.5 y - 12.5$$

$$x = 0.5 y - 12.5 + 20$$

$$x = 0.5 y + 7.5 \rightarrow \text{①}$$

The regression equation y on x

$$(y - \bar{y}) = r \frac{\sigma_y}{\sigma_x} (x - \bar{x})$$

$$(y - 25) = 0.75 \times \frac{3}{2} (x - 20)$$

$$y - 25 = 1.125 (x - 20) \rightarrow y - 25 = 2.25 x$$



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$$y - 25 = 1.125x - 22.5$$

$$y = 1.125x - 22.5 + 25$$

$$y = 1.125x + 2.5 \rightarrow (2)$$

8. Given the following information $\bar{x} = 65$, $\bar{y} = 67$, $\sigma_x = 25$, $\sigma_y = 3.5$, $r = 0.8$ obtain the two regression equation lines the estimate of x , when $y = 70$ and when $x = 58$.

Sol: Regⁿ eqⁿ x on y

$$(x - \bar{x}) = r \frac{\sigma_x}{\sigma_y} (y - \bar{y})$$

$$(x - 65) = 0.8 \times \frac{25}{3.5} (y - 67)$$

$$x - 65 = 5.71 (y - 67)$$

$$x - 65 = 5.71y - 382.57$$

$$x = 5.71y - 382.57 + 65$$

$$x = 5.71y - 317.57 \rightarrow (1)$$

$$\text{when } y = 70 \Rightarrow x = 5.71(70) - 317.57$$

$$x = 399.7 - 317.57$$

$$x = 82.14$$



Regⁿ eqⁿ y on x

$$(y - \bar{y}) = r \frac{S_y}{S_x} (x - \bar{x})$$

$$(y - 67) = \frac{0.8 \times 3.5}{25} (x - 65)$$

$$y - 67 = 0.112 (x - 65)$$

$$y - 67 = 0.112x - 7.28$$

$$y = 0.112x - 7.28 + 67$$

$$y = 0.112x + 59.72 \rightarrow \textcircled{2}$$

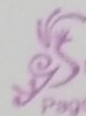
when $x = 58$

$$y = 0.112(58) + 59.72$$

$$y = 6.50 + 59.72$$

$$y = \underline{\underline{66.216}}$$

9. Correlation coefficient between the ages of brothers x and sisters y on a community was found to be 0.8. Average age of the Brothers was $25 = \bar{x}$ and that of Sisters $\bar{y} = 22$ years. There SD were 4 & 5 respectively. Find



1. The expected age of Brother. When Sister's age is 12 years $y = 12$
2. The expected age of Sister. When Brother's age is 33 years. $x = 33$

Sol: Regⁿ eqⁿ x on y

$$\frac{(x - \bar{x})}{s_x} = r \frac{(y - \bar{y})}{s_y}$$

$$(x - 25) = 0.8 \frac{4}{5} (y - 22)$$

$$(x - 25) = 0.64 (y - 22)$$

$$x - 25 = 0.64y - 14.08$$

$$x = 0.64y - 14.08 + 25$$

$$x = 0.64y + 10.92 \rightarrow \textcircled{1}$$

$$\text{When } y = 12$$

$$x = 0.64(12) + 10.92$$

$$x = 7.68 + 10.92$$

$$x = 18.6$$

Regⁿ eqⁿ y on x

$$(y - \bar{y}) = r \frac{s_y}{s_x} (x - \bar{x})$$

$$(y - 22) = 0.8 \times \frac{5}{4} (x - 25)$$

$$y - 22 = 1 (x - 25)$$

$$y - 22 = x - 25$$

$$y = x - 25 + 22$$

$$y = x - 3$$

→ ②

when $x = 33$

$$y = x - 3$$

$$y = 1(33) - 3$$

$$y = 33 - 3$$

$$y = \underline{30}$$